Question 3 (7 marks)

<EFOFEX>
id:fxd{7a6a0f69-1501-49c1-94f6-8175fbf3a95c}

FXData:
</EFOFEX>The diagram shows part of the curve  
with equation

Use integration and the substitution  
 to determine the shaded  
area in the first quadrant bounded by  
 and the curve.

Question 3 (7 marks)

<EFOFEX>
id:fxd{7a6a0f69-1501-49c1-94f6-8175fbf3a95c}

FXData:
</EFOFEX>The diagram shows part of the curve  
with equation

Use integration and the substitution  
 to determine the shaded  
area in the first quadrant bounded by  
 and the curve.

|  |
| --- |
| Solution |
| When and when . |
| Specific behaviours |
| ✓ writes integral in terms of with correct bounds  ü relates and  ü adjusts bounds in terms of  ü writes and simplifies integral in terms of  ü replaces term using trig identity  ü correctly antidifferentiates  ü substitutes and simplifies to obtain area |

Question 7 (6 marks)

<EFOFEX>
id:fxd{38367136-9c86-4a34-baa3-924658b028dc}

FXData:
</EFOFEX>The graph of is shown.

Determine the volume of the solid of revolution  
formed when the part of the curve between  
 and is rotated about the -axis.

Question 7 (6 marks)

<EFOFEX>
id:fxd{38367136-9c86-4a34-baa3-924658b028dc}

FXData:
</EFOFEX>The graph of is shown.

Determine the volume of the solid of revolution  
formed when the part of the curve between  
 and is rotated about the -axis.

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ forms correct integral for volume  ü indicates need for partial fractions and factorises denominator  ü obtains partial fractions  ü rewrites integral using partial fractions  ü antidifferentiates and substitutes  ü correct volume, simplified |

Question 6 (7 marks)

(a) Determine the value of the constant and the value of the constant so that

(2 marks)

(b) Hence use the substitution to determine

(5 marks)

Question 6 (7 marks)

(a) Determine the value of the constant and the value of the constant so that

(2 marks)

|  |
| --- |
| Solution |
| Combining RHS into single fraction and equating numerators:  Hence and . |
| Specific behaviours |
| ✓ indicates appropriate method  ü solves for correctly |

(b) Hence use the substitution to determine

(5 marks)

|  |
| --- |
| Solution |
| Simplify denominator  Also  Hence |
| Specific behaviours |
| ✓ simplifies denominator in terms of  ü correctly relates and  ü uses partial fractions to express integral in terms of  ü obtains antiderivative with absolute value brackets  ü correct result in terms of , and integration constant |

Question 7 (8 marks)

<EFOFEX>
id:fxd{8300724c-5003-4a6a-bc7e-88b9a73b2c5a}

FXData:
</EFOFEX>(a) The line and the curve are  
shown in the diagram. They intersect at the  
origin and at .  
  
Determine the area between the curve and the  
line in the first quadrant.

(5 marks)

(b) Given that , determine . (3 marks)

Question 7 (8 marks)

<EFOFEX>
id:fxd{8300724c-5003-4a6a-bc7e-88b9a73b2c5a}

FXData:
</EFOFEX>(a) The line and the curve are  
shown in the diagram. They intersect at the  
origin and at .  
  
Determine the area between the curve and the  
line in the first quadrant.

(5 marks)

|  |
| --- |
| Solution |
| To determine area under curve let , so that and . Then  Area under line is .  Hence area between is . |
| Specific behaviours |
| ✓ chooses suitable substitution  ü writes integral in terms of  ü obtains antiderivative  ü obtains area under curve  ü obtains area between |

(b) Given that , determine . (3 marks)

|  |
| --- |
| Solution |
| Let , so that , and .  Then |
| Specific behaviours |
| ✓ chooses substitution and relates and  ü writes integral in terms of  ü derives correct value |

Question 1 (5 marks)

(a) Determine . (2 marks)

(b) Evaluate . (3 marks)

Question 1 (5 marks)

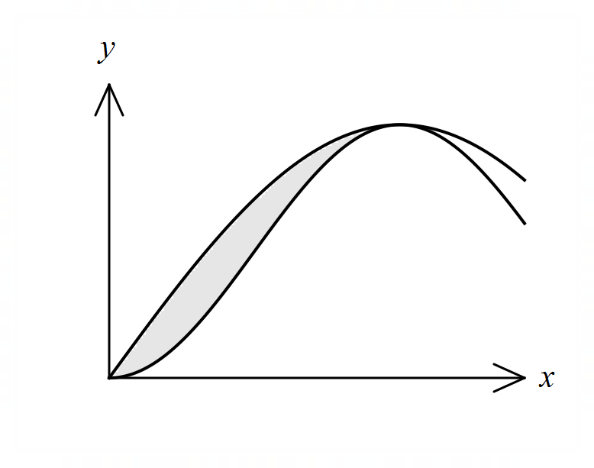
(a) Determine . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ writes in form  ü correct integral with constant |

(b) Evaluate . (3 marks)

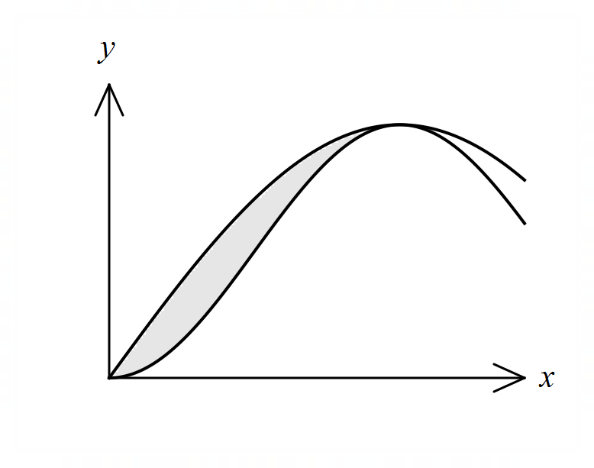
|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ simplifies  ü antiderivative  ü evaluates |

Question 2 (5 marks)

The curves and   
are shown to the right.

Determine the area of the shaded region  
trapped between the curves.

Question 2 (5 marks)

The curves and   
are shown to the right.

Determine the area of the shaded region  
trapped between the curves.

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates bounds  ü writes required integral  ü uses double angle identity  ü antidifferentiates  ü substitutes and simplifies |

Question 6 (6 marks)

(a) Express in the form . (2 marks)

(b) Use the substitution to determine the indefinite integral shown below in the form for . (4 marks)

Question 6 (6 marks)

(a) Express in the form . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates appropriate method  ü correct partial fractions |

(b) Use the substitution to determine the indefinite integral shown below in the form for . (4 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ writes integral in terms of  ü simplifies using result from part (a)  ü obtains antiderivative  ü writes in required form |

Question 1 (6 marks)

The function is defined by

(a) Determine the value of the constant and the value of the constant . (3 marks)

(b) Hence, or otherwise, determine the value of in simplest form. (3 marks)

Question 1 (6 marks)

The function is defined by

(a) Determine the value of the constant and the value of the constant . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ combines fractions correctly   correct value of   correct value of |

(b) Hence, or otherwise, determine the value of in simplest form. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expresses integral using partial fractions   antidifferentiates   substitutes and simplifies |

Question 5 (5 marks)

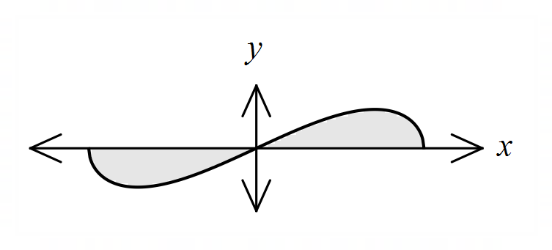
Use the substitution to evaluate

Question 5 (5 marks)

Use the substitution to evaluate

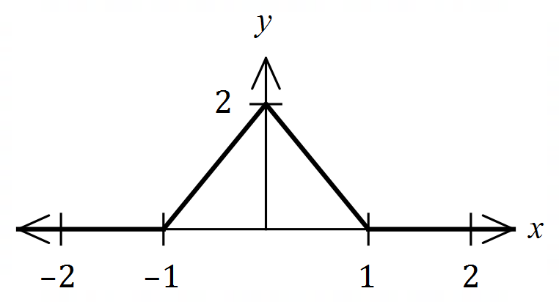
|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ obtains in terms of   changes limits   simplifies integrand   antidifferentiates integrand   evaluates definite integral |

Question 8 (9 marks)

(a) The graph of the curve   
is shown at right.

Using the substitution ,  
or otherwise, determine the area  
between the curve and the -axis.

(6 marks)

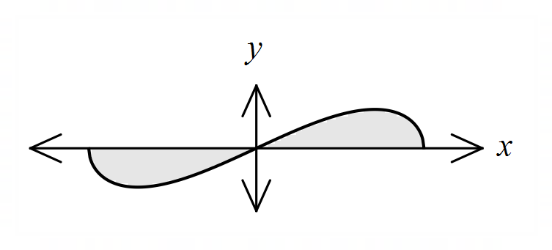


(b) The graph of is shown at right.

Determine the value of .

(3 marks)

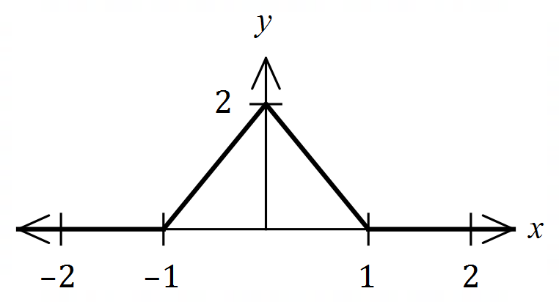
Question 8 (9 marks)

(a) The graph of the curve   
is shown at right.

Using the substitution ,  
or otherwise, determine the area  
between the curve and the -axis.

(6 marks)

|  |
| --- |
| **Solution** |
| Hence . |
| **Specific behaviours** |
| ✓ indicates roots of graph   relates and   integral in terms of   evaluates integral   repeats or uses symmetry to obtain second area   total area |



(b) The graph of is shown at right.

Determine the value of .

|  |
| --- |
| **Solution** |
| Let , so that .  But and so (from graph).  Hence and so |
| **Specific behaviours** |
| ✓ justifies expression for   writes required integral   correct value |

(3 marks)

Question 5 (7 marks)

Determine

(a) . (3 marks)

(b) , using the substitution . (4 marks)

Question 5 (7 marks)

Determine

(a) . (3 marks)

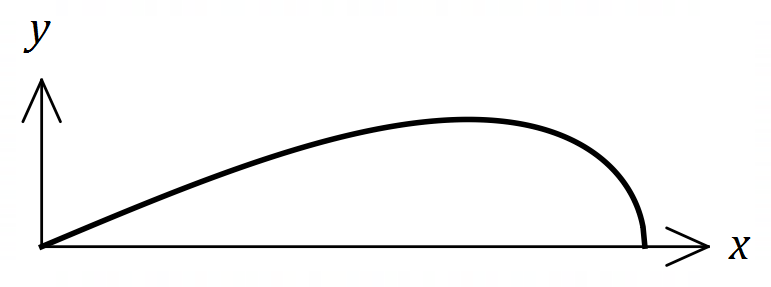
|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates use of partial fractions   correct partial fractions   correct integral, including constant |

(b) , using the substitution . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ relates and   uses trig identity   writes and simplifies integral wrt   integrates and writes wrt |

Question 8 (7 marks)

The graph of in the first quadrant is shown below.

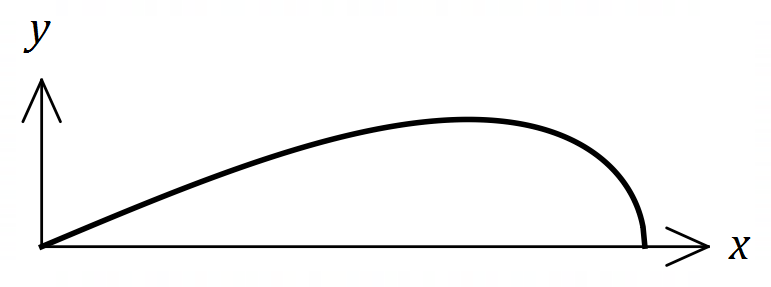


(a) Determine the area of the region shown between the curve and the -axis. (4 marks)

(b) Determine the volume of the solid of revolution formed when the region between the curve, the -axis, and is rotated about the -axis. (3 marks)

Question 8 (7 marks)

The graph of in the first quadrant is shown below.



(a) Determine the area of the region shown between the curve and the -axis. (4 marks)

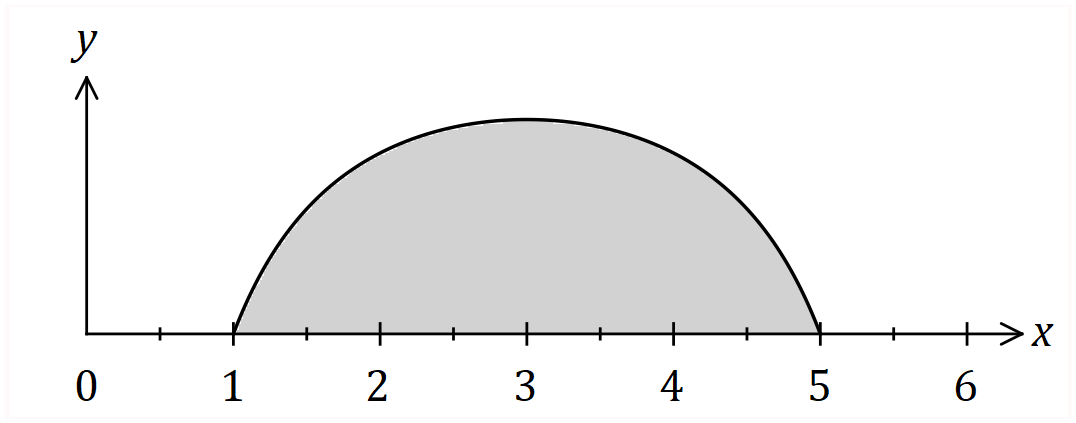
|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct integral in terms of   indicates valid integration method   integrates correctly   simplified area |

(b) Determine the volume of the solid of revolution formed when the region between the curve, the -axis, and is rotated about the -axis. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct integral in terms of   integrates and substitutes correctly   simplified volume |

Question 2 (5 marks)

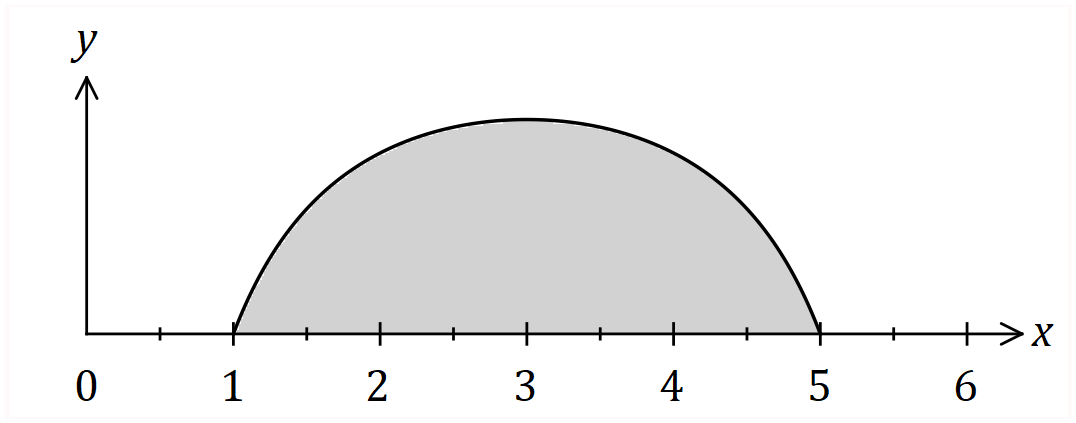
Part of the graph of is shown below.



Determine the shaded area, bounded by the curve and the -axis.

Question 2 (5 marks)

Part of the graph of is shown below.

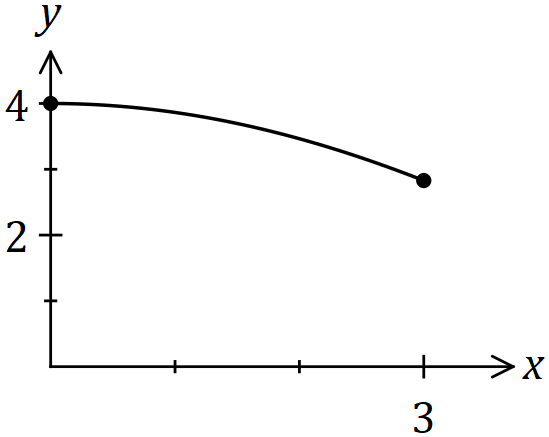


Determine the shaded area, bounded by the curve and the -axis.

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
|  integral, recognising need for partial fractions   obtains partial fractions   integrates   substitutes limits of integration   simplifies until just one logarithm remains |

Question 3 (4 marks)

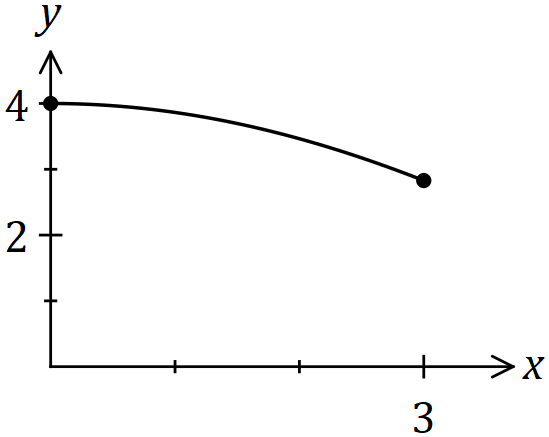
The curve defined by , where , is shown below.



Determine the volume of the solid generated when the area bounded by the axis and the curve is rotated about the axis between and .

Question 3 (4 marks)

The curve defined by , where , is shown below.



Determine the volume of the solid generated when the area bounded by the axis and the curve is rotated about the axis between and .

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes integral   re-writes integral using double angle identity   integrates   substitutes both bounds and simplifies |

Question 3 (7 marks)

(a) Determine the slope of the graph of at the point . (3 marks)

(b) Use the substitution to express in terms of and hence evaluate the integral. (4 marks)

Question 3 (7 marks)

(a) Determine the slope of the graph of at the point . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ differentiates LHS  ✓ differentiates RHS  ✓ substitutes and simplifies |

(b) Use the substitution to express in terms of and hence evaluate the integral. (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expresses integrand in terms of  ✓ changes limits  ✓ antidifferentiates  ✓ evaluates |